

On soft topological space via semiopen and semiclosed soft sets

J. Mahanta, P. K. Das

Department of Mathematics
NERIST, Nirjuli
Arunachal Pradesh, 791 109, INDIA.

Abstract

This paper introduces semiopen and semiclosed soft sets in soft topological spaces. The notions of interior and closure are generalized using these sets. A detail study is carried out on properties of semiopen, semiclosed soft sets, semi interior and semi closure of a soft set in a soft topological space. Various forms of soft functions, like semicontinuous, irresolute, semiopen soft functions are introduced and characterized. Further soft semicompactness, soft semiconnectedness and soft semiseparation axioms are introduced and studied.

Keywords:

Soft topological space, semiopen soft set, soft semicompactness, soft semicontinuity, soft semiconnectedness.
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1. Introduction and Preliminaries

Soft set was introduced by Molodtsov [1] in the year 1999. The notion of topological space for soft sets was formulated by Shabir et. al. [3]. Of late many authors have studied various properties of soft topological spaces.

This paper aims to introduce and give a detail study of semiopen soft set, semiclosed soft set, semicontinuity, semicompactness, semiconnectedness and semiseparation axioms.

Here are some definitions and results required in the sequel.

Definition 1.1. [3] Let τ be a collection of soft sets over a universe U with a fixed set A of parameters, then $\tau \subseteq SS(U)_A$ is called a soft topology on U with a fixed set A if

- (i) Φ_A, U_A belongs to τ ;
- (ii) the union of any number of soft sets in τ belongs to τ ;
- (iii) the intersection of any two soft sets in τ belongs to τ .

Then (U_A, τ) is called a soft topological space over U .

Definition 1.2. [4] A soft basis of a soft topological space (U_A, τ) is a subcollection \mathcal{B} of open soft sets such that every element of τ can be expressed as the union of elements of \mathcal{B} .

Definition 1.3. [4] Let (U_A, τ) be a soft topological space and $U_B \subseteq U_A$. then the collection $\tau_{U_B} = \{U_{A_i} \cap U_B \mid U_{A_i} \in \tau, i \in I \subseteq \mathbb{N}\}$ is called a soft subspace topology on U_B .

Definition 1.4. [2] A soft set $F_A \in SS(U)_A$ is called a soft point in U_A , denoted by e_F , if for each element $e \in A$, $F(e) \neq \Phi$ and $F(e') = \Phi$ for all $e' \in A - \{e\}$.

Definition 1.5. [2] A soft point e_F is said to be in the soft set G_A , denoted by $e_F \tilde{\in} G_A$, if for some element $e \in A$ and $F(e) = G(e)$.

Email addresses: jm_nerist@yahoo.in (J. Mahanta), pkd_ma@yahoo.cam (P. K. Das)

Definition 1.6. [2] A family Ψ of soft sets is a cover of a soft set F_A if $F_A \subseteq \bigcup \{(F_i)_A \mid (F_i)_A \in \Psi, i \in I\}$.
A subcover of Ψ is a subfamily of Ψ which is also a cover.

Definition 1.7. [2] A family Ψ of soft sets has the finite intersection property (FIP) if the intersection of the members of each finite subfamily of Ψ is not null soft set.

Throughout this study, F_A denotes a soft set, (U_A, τ) denotes a soft topological space.

2. Semiopen and semiclosed soft sets

In this section, we introduce semiopen and semiclosed soft sets and study various notions related to this structure.

Definition 2.1. In a soft topological space (U_A, τ) , a soft set

- (i) G_A is said to be semiopen soft set if \exists an open soft set H_A such that $H_A \subseteq G_A \subseteq \overline{H_A}$;
- (ii) F_A is said to be semiclosed soft set if \exists a closed soft set K_A such that $K_A^0 \subseteq F_A \subseteq K_A$;

Example 2.2. Consider the soft topological spaces (U_A, τ) as defined in Example 3 of [3]. Here $G_A(e_1) = \{h_1, h_2\}$, $G_A(e_2) = \{h_1\}$ is a semiopen soft set, as F_{1A} is a open soft set such that $F_{1A} \subseteq G_A \subseteq \overline{F_{1A}} = F_{1A}$.

$K_A(e_1) = \{h_3\}$, $K_A(e_2) = \{h_3\}$ is a semiclosed soft set, as $(F_{1A})^c$ is a closed soft set such that $((F_{1A})^c)^0 \subseteq K_A \subseteq (F_{1A})^c$.

Remark 2.3. Every open (closed) soft set is a semiopen (semiclosed) soft set but not conversely.

Remark 2.4. Φ_A and U_A are always semiclosed and semiopen.

From now onwards, we shall denote the family of all semiopen soft sets (semiclosed soft sets) of a soft topological space (U_A, τ) by $S OSS(U)_A$ ($SCSS(U)_A$).

Theorem 2.5. Arbitrary union of semiopen soft sets is a semiopen soft set.

Proof. Let $\{(G_\lambda)_A \mid \lambda \in \Lambda\}$ be a collection of semiopen soft sets of a soft topological space (U_A, τ) . Then \exists an open soft sets $(H_\lambda)_A$ such that $(H_\lambda)_A \subseteq (G_\lambda)_A \subseteq \overline{(H_\lambda)_A}$ for each λ ; hence $\bigcup (H_\lambda)_A \subseteq \bigcup (G_\lambda)_A \subseteq \bigcup \overline{(H_\lambda)_A}$ and $\bigcup (H_\lambda)_A$ is open soft set. So, it is concluded that $\bigcup (G_\lambda)_A$ is a semiopen soft set. \square

Remark 2.6. Arbitrary intersection of semiclosed soft sets is a semiclosed soft set.

Theorem 2.7. If a semiopen soft set G_A is such that $G_A \subseteq K_A \subseteq \overline{G_A}$, then K_A is also semiopen.

Proof. As G_A is semiopen soft set \exists an open soft set H_A such that $H_A \subseteq G_A \subseteq \overline{H_A}$; then by hypothesis $H_A \subseteq K_A$ and $\overline{G_A} \subseteq \overline{H_A} \Rightarrow K_A \subseteq \overline{G_A} \subseteq \overline{H_A}$ i.e., $H_A \subseteq K_A \subseteq \overline{H_A}$, hence K_A is a semiopen soft set. \square

Theorem 2.8. If a semiclosed soft set F_A is such that $(F_A)^0 \subseteq K_A \subseteq F_A$, then K_A is also semiclosed.

Theorem 2.9. A soft set $G_A \in S OSS(U)_A \Leftrightarrow$ for every soft point $e_G \in G_A$, \exists a soft set $H_A \in S OSS(U)_A$ such that $e_G \in H_A \subseteq G_A$.

Proof. (\Rightarrow) Take $H_A = G_A$.

$$(\Leftarrow) G_A = \bigcup_{e_G \in G_A} (e_G) \subseteq \bigcup_{e_G \in G_A} H_A \subseteq G_A.$$

\square

Definition 2.10. Let (U_A, τ) be a soft topological space and G_A be a soft set over U .

- (i) The soft semi closure of G_A is a soft set
 $ssclG_A = \bigcap \{S_A \mid G_A \subseteq S_A \text{ and } S_A \in SCSS(U)_A\};$
- (ii) The soft semi interior of G_A is a soft set
 $ssintG_A = \bigcup \{S_A \mid S_A \subseteq G_A \text{ and } S_A \in SOS S(U)_A\}.$

$ssclG_A$ is the smallest semiclosed soft set containing G_A and $ssintG_A$ is the largest semiopen set contained in G_A .

Theorem 2.11. Let (U_A, τ) be a soft topological space and G_A and K_A be two soft sets over U , then

- (i) $G_A \in SCSS(U)_A \Leftrightarrow G_A = ssclG_A;$
- (ii) $G_A \in SOS S(U)_A \Leftrightarrow G_A = ssintG_A;$
- (iii) $(ssclG_A)^c = ssint(G_A^c);$
- (iv) $(ssintG_A)^c = sscl(G_A^c);$
- (v) $G_A \subseteq K_A \Rightarrow ssintG_A \subseteq ssintK_A;$
- (vi) $G_A \subseteq K_A \Rightarrow ssclG_A \subseteq ssclK_A;$
- (vii) $sscl\Phi_A = \Phi_A$ and $ssclU_A = U_A;$
- (viii) $ssint\Phi_A = \Phi_A$ and $ssintU_A = U_A;$
- (ix) $sscl(G_A \tilde{\cup} K_A) = ssclG_A \tilde{\cup} ssclK_A;$
- (x) $ssint(G_A \tilde{\cap} K_A) = ssintG_A \tilde{\cap} ssintK_A;$
- (xi) $sscl(G_A \tilde{\cap} K_A) \subseteq ssclG_A \tilde{\cap} ssclK_A;$
- (xii) $ssint(G_A \tilde{\cup} K_A) \subseteq ssintG_A \tilde{\cup} ssintK_A;$
- (xiii) $sscl(ssclG_A) = ssclG_A;$
- (xiv) $ssint(ssintG_A) = ssintG_A.$

Proof. Let G_A and K_A be two soft sets over U .

- (i) Let G_A be a semiclosed soft set. Then it is the smallest semiclosed set containing itself and hence $G_A = ssclG_A$.
 On the other hand, let $G_A = ssclG_A$ and $ssclG_A \in SCSS(U) \Rightarrow G_A \in SCSS(U)$.
- (ii) Similar to (i).
- (iii)

$$\begin{aligned}
 (ssclG_A)^c &= (\bigcap \{S_A \mid G_A \subseteq S_A \text{ and } S_A \in SCSS(U)_A\})^c \\
 &= \bigcup \{S_A^c \mid G_A \subseteq S_A \text{ and } S_A \in SCSS(U)_A\} \\
 &= \bigcup \{S_A^c \mid S_A^c \subseteq G_A^c \text{ and } S_A^c \in SOS S(U)_A\} \\
 &= ssint(G_A^c).
 \end{aligned}$$

- (iv) Similar to (iii).
- (v) Follows from definition.
- (vi) Follows from definition.
- (vii) Since Φ_A and U_A are semiclosed soft sets so $sscl\Phi_A = \Phi_A$ and $ssclU_A = U_A$.
- (viii) Since Φ_A and U_A are semiopen soft sets so $ssint\Phi_A = \Phi_A$ and $ssintU_A = U_A$.
- (ix) We have $G_A \subseteq G_A \tilde{\cup} K_A$ and $K_A \subseteq G_A \tilde{\cup} K_A$. Then by (vi), $ssclG_A \subseteq sscl(G_A \tilde{\cup} K_A)$ and $ssclK_A \subseteq sscl(G_A \tilde{\cup} K_A) \Rightarrow ssclK_A \tilde{\cup} ssclG_A \subseteq sscl(G_A \tilde{\cup} K_A)$.
 Now, $ssclG_A, ssclK_A \in SCSS(U)_A \Rightarrow ssclG_A \tilde{\cup} ssclK_A \in SCSS(U)_A$.
 Then $G_A \subseteq ssclG_A$ and $K_A \subseteq ssclK_A$ imply $G_A \tilde{\cup} K_A \subseteq ssclG_A \tilde{\cup} ssclK_A$. i.e., $ssclG_A \tilde{\cup} ssclK_A$ is a semiclosed set containing $G_A \tilde{\cup} K_A$. But $sscl(G_A \tilde{\cup} K_A)$ is the smallest semiclosed soft set containing $G_A \tilde{\cup} K_A$. Hence $sscl(G_A \tilde{\cup} K_A) \subseteq ssclG_A \tilde{\cup} ssclK_A$. So, $sscl(G_A \tilde{\cup} K_A) = ssclG_A \tilde{\cup} ssclK_A$.

- (x) Similar to (ix).
- (xi) We have $G_A \widetilde{\cap} K_A \subseteq G_A$ and $G_A \widetilde{\cap} K_A \subseteq K_A$
 $\Rightarrow sscl(G_A \widetilde{\cap} K_A) \subseteq sscl G_A$ and $sscl(G_A \widetilde{\cap} K_A) \subseteq sscl K_A$
 $\Rightarrow sscl(G_A \widetilde{\cap} K_A) \subseteq sscl G_A \cap sscl K_A$.
- (xii) Similar to (xi).
- (xiii) Since $sscl G_A \in SCSS(U)$ so by (i), $sscl(sscl G_A) = sscl G_A$.
- (xiv) Since $ssint G_A \in SOS S(U)$ so by (ii), $ssint(ssint G_A) = ssint G_A$.

□

Theorem 2.12. *If G_A is any soft set in a soft topological space (U_A, τ) then following are equivalent:*

- (i) G_A is semiclosed soft set;
- (ii) $(\overline{G_A})^0 \subseteq G_A$;
- (iii) $(G_A^c)^0 \supseteq G_A^c$.
- (iv) G_A^c is semiopen soft set;

Proof. (i) \Rightarrow (ii) If G_A is semiclosed soft set, then \exists closed soft set H_A such that $H_A^0 \subseteq G_A \subseteq H_A \Rightarrow H_A^0 \subseteq G_A \subseteq \overline{G_A} \subseteq H_A$. By the property of interior we then have $(\overline{G_A})^0 \subseteq H_A^0 \subseteq G_A$;

(ii) \Rightarrow (iii) $(\overline{G_A})^0 \subseteq G_A \Rightarrow G_A^c \subseteq ((\overline{G_A})^0)^c = (\overline{G_A^c})^0 \supseteq G_A^c$.

(iii) \Rightarrow (iv) $H_A = (G_A^c)^0$ is an open soft set such that $(G_A^c)^0 \subseteq G_A^c \subseteq (\overline{G_A^c})^0$, hence G_A^c is semiopen.

(iv) \Rightarrow (i) As G_A^c is semiopen \exists an open soft set H_A such that $H_A \subseteq G_A^c \subseteq \overline{H_A} \Rightarrow H_A^c$ is a closed soft set such that $G_A \subseteq H_A^c$ and $G_A^c \subseteq \overline{H_A} \Rightarrow (H_A^c)^0 \subseteq G_A$, hence G_A is semiclosed soft set. □

3. Soft Semicontinuous, Soft Irresolute, Soft Semiopen and Soft Semoclosed Functions

Here we introduce different types of soft functions in soft topological spaces and investigate their properties.

Definition 3.1. *Let (U_A, τ) and (U_B, δ) be two soft topological spaces. A soft function $f : U_A \rightarrow U_B$ is said to be*

- (i) *soft semicontinuous if for each soft open set G_B of U_B , the inverse image $f^{-1}(G_B)$ is soft semiopen set of U_A ;*
- (ii) *soft irresolute if for each soft semiopen set G_B of U_B , the inverse image $f^{-1}(G_B)$ is soft semiopen set of U_A ;*
- (iii) *soft semiopen function if for each open soft set G_A of U_A , the image $f(G_A)$ is semiopen soft set of U_B ;*
- (iv) *soft semiclosed function if for each closed soft set F_A of U_A , the image $f(F_A)$ is semiclosed soft set of U_B .*

Remark 3.2. (a) A soft function $f : U_A \rightarrow U_B$ is soft semicontinuous if for each soft closed set F_B of U_B , the inverse image $f^{-1}(F_B)$ is soft semiclosed set of U_A .

(b) A soft semicontinuous function is soft irresolute.

Theorem 3.3. *A soft function $f : U_A \rightarrow U_B$ is soft semicontinuous iff $f(sscl F_A) \subseteq \overline{f(F_A)}$ for every soft set F_A of U_A .*

Proof. Let $f : U_A \rightarrow U_B$ is soft semicontinuous. Now $\overline{f(F_A)}$ is a soft closed set of U_B , so by soft semicontinuity of f , $f^{-1}(\overline{f(F_A)})$ is soft semiclosed and $F_A \subseteq f^{-1}(\overline{f(F_A)})$. But $sscl F_A$ is the smallest semiclosed set containing F_A , hence $sscl F_A \subseteq f^{-1}(\overline{f(F_A)}) \Rightarrow f(sscl F_A) \subseteq \overline{f(F_A)}$.

Conversely, let F_B be any soft closed set of $U_B \Rightarrow f^{-1}(F_B) \in U_A \Rightarrow f(sscl(f^{-1}(F_B))) \subseteq \overline{f(f^{-1}(F_B))} \Rightarrow f(sscl(f^{-1}(F_B))) \subseteq \overline{F_B} = F_B \Rightarrow sscl(f^{-1}(F_B)) = f^{-1}(F_B)$, hence is semiclosed. □

Theorem 3.4. *A soft function $f : U_A \rightarrow U_B$ is soft semicontinuous iff $f^{-1}(H_B)^0 \subseteq ssint(f^{-1}(H_B))$ for every soft set H_B of U_B .*

Proof. Let $f : U_A \rightarrow U_B$ is soft semicontinuous. Now $(f(G_A))^0$ is a soft open set of U_B , so by soft semicontinuity of f , $f^{-1}(f(G_A))^0$ is soft semiopen and $f^{-1}(f(G_A))^0 \subseteq G_A$. As $ssint G_A$ is the largest soft semiopen set contained in G_A , $f^{-1}(f(G_A))^0 \subseteq ssint G_A$.

Conversely, take a soft open set $G_B \Rightarrow f^{-1}(G_B)^0 \subseteq ssint(f^{-1}(G_B)) \Rightarrow f^{-1}(G_B) \subseteq ssint(f^{-1}(G_B)) \Rightarrow f^{-1}(G_B)$ is soft semiopen.

□

Theorem 3.5. A soft function $f : U_A \rightarrow U_B$ is soft semiopen iff $f((F_A)^0) \subseteq ssint(f(F_A))$ for every soft set F_A of U_A .

Proof. If $f : U_A \rightarrow U_B$ is soft semiopen, then $f((F_A)^0) = ssint f((F_A)^0) \subseteq ssint f(F_A)$.

On the other hand, take a soft open set G_A of U_A . Then by hypothesis, $f(G_A) = f((G_A)^0) \subseteq ssint(f(G_A)) \Rightarrow f(G_A)$ is soft semiopen in U_B .

□

Theorem 3.6. Let $f : U_A \rightarrow U_B$ be soft semiopen. If K_B is a soft set and F_A is closed soft set containing $f^{-1}(K_B)$ then \exists a semiclosed soft set H_B such that $K_B \subseteq H_B$ and $f^{-1}(H_B) \subseteq F_A$.

Proof. Take $H_B = (f(F_A^c))^c$. Now $f^{-1}(K_B) \subseteq F_A \Rightarrow f(F_A^c) \subseteq K_B^c$. Then $F_A^c \text{ open} \Rightarrow f(F_A^c)$ is semiopen, so H_B is semiclosed and $K_B \subseteq H_B$ and $f^{-1}(H_B) \subseteq F_A$.

□

Theorem 3.7. A soft function $f : U_A \rightarrow U_B$ is soft semiclosed iff $sscl f(F_A) \subseteq f(\overline{F_A})$ for every soft set F_A of U_A .

4. Semicompact soft topological spaces

This section is devoted to introduce semicompactness in soft topological spaces along with characterization of semicompact soft topological spaces.

Definition 4.1. A cover of a soft set is said to be a semiopen soft cover if every member of the cover is a semiopen soft set.

Definition 4.2. A soft topological space (U_A, τ) is said to be semicompact if each semiopen soft cover of U_A has a finite subcover.

Remark 4.3. Every compact soft space is also semicompact.

Theorem 4.4. A soft topological space (U_A, τ) is semicompact \Leftrightarrow each family of semiclosed soft sets with the FIP has a nonempty intersection.

Proof. Let $\{(F_A)_\lambda \mid \lambda \in \Lambda\}$ be a collection of semiclosed soft sets with the FIP. If possible, assume $\bigcap_{\lambda \in \Lambda} (F_A)_\lambda = \Phi_A \Rightarrow \bigcup_{\lambda \in \Lambda} ((F_A)_\lambda)^c = U_A$. So, the collection $\{((F_A)_\lambda)^c \mid \lambda \in \Lambda\}$ forms a soft semiopen cover of U_A , which is semicompact. So, \exists a finite subcollection Δ of Λ which also covers U_A . i.e., $\bigcup_{\lambda \in \Delta} ((F_A)_\lambda)^c = U_A \Rightarrow \bigcap_{\lambda \in \Delta} (F_A)_\lambda = \Phi_A$, a contradiction.

For the converse, if possible, let (U_A, τ) be not semicompact. Then \exists a semiopen cover $\{(G_A)_\lambda \mid \lambda \in \Lambda\}$ of U_A , such that for every finite subcollection Δ of Λ we have $\bigcup_{\lambda \in \Delta} (G_A)_\lambda \neq U_A \Rightarrow \bigcap_{\lambda \in \Delta} ((G_A)_\lambda)^c \neq \Phi_A$. Hence $\{((G_A)_\lambda)^c \mid \lambda \in \Lambda\}$ has the FIP. So, by hypothesis $\bigcap_{\lambda \in \Lambda} ((G_A)_\lambda)^c \neq \Phi_A \Rightarrow \bigcup_{\lambda \in \Lambda} (G_A)_\lambda \neq U_A$, a contradiction.

□

Theorem 4.5. A soft topological space (U_A, τ) is semicompact iff every family Ψ of soft sets with the FIP, $\bigcap_{G_A \in \Psi} sscl G_A \neq \Phi_A$.

Proof. Let (U_A, τ) be semicompact and if possible let $\bigcap_{G_A \in \Psi} sscI G_A = \Phi_A$ for some family Ψ of soft sets with the FIP. So, $\bigcup_{G_A \in \Psi} (sscI G_A)^c = U_A \Rightarrow \Upsilon = \{(sscI G_A)^c \mid G_A \in \Psi\}$ is a semiopen cover of U_A . Then by semicompactness of U_A , \exists a finite subcover ω of Υ . i.e., $\bigcup_{G_A \in \omega} (sscI G_A)^c = U_A \Rightarrow \bigcup_{G_A \in \omega} G_A^c = U_A \Rightarrow \bigcap_{G_A \in \omega} G_A = \Phi_A$, a contradiction. Hence $\bigcap_{G_A \in \Psi} sscI G_A \neq \Phi_A$.

Conversely, we have $\bigcap_{G_A \in \Psi} sscI G_A \neq \Phi_A$, for every family Ψ of soft sets with FIP. Assume (U_A, τ) is not semicompact. Then \exists a family Υ of semiopen soft sets covering U without a finite subcover. So for every finite subfamily ω of Υ we have $\bigcup_{G_A \in \omega} G_A \neq U_A \Rightarrow \bigcap_{G_A \in \omega} G_A^c \neq \Phi_A \Rightarrow \{G_A^c \mid G_A \in \Upsilon\}$ is a family of soft sets with FIP. Now $\bigcup_{G_A \in \Upsilon} G_A = U_A \Rightarrow \bigcap_{G_A \in \Upsilon} G_A^c = \Phi_A \Rightarrow \bigcap_{G_A \in \Upsilon} sscI(G_A^c) = \Phi_A$, a contradiction. \square

Theorem 4.6. *Semicontinuous image of a soft semicompact space is soft compact.*

Proof. Let $f : U_A \rightarrow U_B$ be a semicontinuous function from a semicompact soft topological space (U_A, τ) to (U_B, δ) . Take a soft open cover $\{(G_B)_\lambda \mid \lambda \in \Lambda\}$ of $U_B \Rightarrow \{f^{-1}((G_B)_\lambda) \mid \lambda \in \Lambda\}$ forms a soft semiopen cover of $U_A \Rightarrow \exists$ a finite subset Δ of Λ such that $\{f^{-1}((G_B)_\lambda) \mid \lambda \in \Delta\}$ forms a semiopen cover of $U_A \Rightarrow \{(G_B)_\lambda \mid \lambda \in \Delta\}$ forms a finite soft opencover of U_B . \square

Theorem 4.7. *Semiclosed subspace of a semicompact soft topological space is soft semicompact.*

Proof. Let U_B a semiclosed subspace of a semicompact soft topological space (U_A, τ) and $\{(G_B)_\lambda \mid \lambda \in \Lambda\}$ be a semiopen cover of $U_B \Rightarrow$ for each $(G_B)_\lambda$, \exists a semiopen soft set G_A of U_A such that $G_B = G_A \tilde{\cap} U_B$. Then the family $\{(G_B)_\lambda \mid \lambda \in \Lambda\} \cup (U_A - U_B)$ is a soft semi open cover of U_A , which has a finite subcover. So $\{(G_B)_\lambda \mid \lambda \in \Lambda\}$ has a finite subfamily to cover U_B . Hence U_B is semicompact. \square

5. Semi Connectedness in soft topological spaces

In this section, we introduce and study the notion of semi connectedness in a soft topological space.

Definition 5.1. *Two soft sets F_A and G_B are said to be disjoint if $A \cap B = \emptyset$ and $F(a) \cap G(b) = \emptyset, \forall a \in A, b \in B$.*

Definition 5.2. *A soft semiseparation of soft topological space (U_A, τ) is a pair F_A, G_A of disjoint nonnull semiopen sets whose union is U_A .*

If there doesn't exist a soft semiseparation of U_A , then the soft topological space is said to be soft semiconnected, otherwise soft semidisconnected

Theorem 5.3. *If the soft sets H_A and G_A form a soft semiseparation of U_A , and if $V_B, B \subset A$ is a soft semiconnected subspace of U_A , then $V_B \tilde{\subset} H_A$ or $V_B \tilde{\subset} G_A$.*

Proof. Since H_A and G_A are disjoint semiopen soft sets, so are $H_A \tilde{\cap} V_B$ and $G_A \tilde{\cap} V_B$ and their soft union gives V_B , i.e. they would constitute a soft semiseparation of V_B , a contradiction. Hence, one of $H_A \tilde{\cap} V_B$ and $G_A \tilde{\cap} V_B$ is empty and so V_B is entirely contained in one of them. \square

Theorem 5.4. *Let V_A be a soft semiconnected subspace of U_A . If $V_A \tilde{\subset} K_A \tilde{\subset} \overline{V_A}$, then K_A is also soft semiconnected.*

Proof. Let the soft set K_A satisfies the hypothesis. If possible, let F_A and G_A form a soft semiseparation of K_A . Then by theorem 5.3, $V_A \tilde{\subset} F_A$ or $V_A \tilde{\subset} G_A$. Let $V_A \tilde{\subset} F_A \Rightarrow sscI(V_A) \tilde{\subset} sscI F_A$; since $sscI F_A$ and G_A are disjoint, V_A cannot intersect G_A . This contradicts the fact that G_A is a nonempty subset of V_A . \square

Theorem 5.5. A soft topological space (U_A, τ) is semidisconnected $\Leftrightarrow \exists$ a nonnull proper soft subset of U_A which is both semiopen and semiclosed.

Proof. Let K_A be a nonnull proper soft subset of U_A which is both semiopen and semiclosed. Now $H_A = (K_A)^c$ is nonnull proper subset of U_A which is also both semiopen and semiclosed $\Rightarrow sscl K_A = K_A$ and $sscl H_A = H_A \Rightarrow U_A$ can be expressed as the soft union of two semiseparated soft sets K_A, H_A and so is semidisconnected.

Conversely, let U_A be semidisconnected $\Rightarrow \exists$ nonnull soft subsets K_A and H_A such that $sscl K_A \tilde{\cap} H_A = \Phi$, $K_A \tilde{\cap} sscl H_A = \Phi$ and $K_A \tilde{\cup} H_A = U_A$. Now $K_A \tilde{\subseteq} sscl K_A$ and $sscl K_A \tilde{\cap} H_A = \Phi_A \Rightarrow K_A \tilde{\cap} H_A = \Phi_A \Rightarrow H_A = (K_A)^c$. Then $K_A \tilde{\cup} sscl H_A = U_A$ and $K_A \tilde{\cap} sscl H_A = \Phi_A \Rightarrow K_A = (sscl H_A)^c$ and similarly $H_A = (sscl K_A)^c \Rightarrow K_A, H_A$ are semiopen sets being the complements of semiclosed soft sets. Also $H_A = (K_A)^c \Rightarrow$ they are also semiclosed. \square

Theorem 5.6. Semicontinuous image of a soft semiconnected soft topological space is soft connected.

Proof. Let $f : U_A \rightarrow V_B$ be a semicontinuous function from a semiconnected soft topological space (U_A, τ) to a soft topological space (V_B, δ) . It suffices to consider the surjective function $g : U_A \rightarrow f(U_A)$. Suppose $f(U_A) = K_B \tilde{\cup} H_B$ be a soft separation. i.e., K_B and H_B are disjoint soft open sets whose union is $f(U_A) \Rightarrow f^{-1}(K_B)$ and $f^{-1}(H_B)$ are disjoint soft semiopen sets whose union is U_A . So, $f^{-1}(K_B)$ and $f^{-1}(H_B)$ form a soft semiseparation of U_A , a contradiction. \square

Theorem 5.7. Irresolute image of a soft semiconnected soft topological space is soft semiconnected.

6. Semi Separation Axioms

Here we consider different types of separation axioms for a soft topological space using semiopen and semiclosed soft sets.

Definition 6.1. A soft topological space (U_A, τ) is said to be a soft semi T_0 - space if for two disjoint soft points e_G, e_F, \exists a semiopen set containing one but not the other.

Example 6.2. A discrete soft topological space is a soft semi T_0 - space since every $e_F \in U$ is a semiopen soft set in the discrete space.

Theorem 6.3. A soft subspace of a soft semi T_0 - space is soft semi T_0 .

Proof. Let V_B be a soft subspace of a soft semi T_0 - space U_A and let e_F, e_G be two distinct soft points of V_B . Then these soft points are also in $U_A \Rightarrow \exists$ a semiopen soft set H_A containing one soft point but not the other $\Rightarrow H_A \tilde{\cap} V_B$ is a semiopen soft set containing one soft point but not the other. \square

Definition 6.4. A soft topological space (U_A, τ) is said to be a soft semi T_1 - space if for two distinct soft points $e_F, e_G \in U_A, \exists$ soft semiopen sets H_A and G_A such that

$e_F \tilde{\in} H_A$ and $e_G \tilde{\notin} H_A$;
 $e_G \tilde{\in} G_A$ and $e_F \tilde{\notin} G_A$.

Theorem 6.5. If every soft point of a soft topological space (U_A, τ) is a semiclosed soft set then (U_A, τ) is a soft semi T_1 - space.

Proof. Let e_F be a soft point of U_A which is a semiclosed soft set $\Rightarrow (e_F)^c$ is a semiopen soft set. Then for distinct soft points e_F, e_G , we have $(e_F)^c, (e_G)^c$ are semiopen soft sets such that $e_G \tilde{\in} (e_F)^c$ and $e_G \tilde{\notin} e_F$; $e_F \tilde{\in} (e_G)^c$ and $e_F \tilde{\notin} e_G$. \square

Theorem 6.6. A soft subspace of a soft semi T_1 - space is soft semi T_1 .

Definition 6.7. A soft topological space (U_A, τ) is said to be a soft semi T_2 -space if and only if for distinct soft points $e_F, e_G \in U_A$, \exists disjoint soft semiopen sets H_A and G_A such that $e_F \in H_A$ and $e_G \in G_A$.

Theorem 6.8. A soft subspace of a soft semi T_2 -space is soft semi T_2 .

Proof. Let (U_A, τ) be a soft semi T_2 -space and V_B be a soft subspace of U_A , where $B \subset A$ and $V \subset U$. Let e_F and e_G be two distinct soft points of U_B . U_A is soft semi $T_2 \Rightarrow \exists$ two disjoint soft semiopen sets H_A and G_A such that $e_F \in H_A$, $e_G \in G_A$. Then $H_A \cap U_B$ and $G_A \cap U_B$ are semiopen soft sets satisfying the requirements for U_B to be a soft semi T_2 -space. \square

Definition 6.9. A soft topological space (U_A, τ) is said to be a soft semiregular space if for every soft point e_K and semiclosed soft set F_A not containing e_K , \exists disjoint soft semiopen sets G_{A_1}, G_{A_2} such that $e_K \in G_{A_1}$ and $F_A \subseteq G_{A_2}$.
A soft semiregular T_1 -space is called a soft semi T_3 -space,

Remark 6.10. It can be shown that the property of being soft semi T_3 is hereditary.

Remark 6.11. Every soft semi T_3 -space is soft semi T_2 -space, every soft semi T_2 -space is soft semi T_1 -space and every soft semi T_1 -space is soft semi T_0 -space.

Theorem 6.12. Every soft semicompact semi T_2 -space is soft semi T_3 .

Proof. It suffices to show every semicompact soft topological space is semiregular. Let e_L be a soft point and F_A be a semiclosed soft set not containing the point $\Rightarrow e_L \in (F_A)^c$. Now for each soft point e_F , \exists disjoint semiopen soft sets $(K_A)_1$ and $(H_A)_1$ such that $e_F \in (K_A)_1$ and $e_L \in (H_A)_1$. So the collection $\{(K_A)_\lambda \mid \lambda \in \Lambda\}$ forms a semiopen cover of F_A . Now F_A is a semiclosed soft set $\Rightarrow F_A$ is semicompact. Hence \exists a finite subfamily Δ of Λ such that $F_A \subseteq \bigcup_{\lambda \in \Delta} (K_A)_\lambda$.

Take $H_A = \bigcap_{i=1}^n (H_A)_i$ and $K_A = \bigcup_{i=1}^n (K_A)_i$. Then H_A, K_A are disjoint semiopen sets such that e_L is a soft point of H_A and $F_A \subseteq K_A$. \square

Definition 6.13. A soft topological space (U_A, τ) is said to be a soft seminormal space if for every pair of disjoint semiclosed soft sets F_A and K_A , \exists two disjoint soft semiopen sets H_{A_1}, H_{A_2} such that $F_A \subseteq H_{A_1}$ and $K_A \subseteq H_{A_2}$.
A soft seminormal T_1 -space is called a soft semi T_4 -space.

Remark 6.14. Every soft semi T_4 -space is soft semi T_3 .

Theorem 6.15. A soft topological space (U_A, τ) is seminormal iff for any semiclosed soft set F_A and semiopen soft set G_A containing F_A , there exists an semiopen soft set H_A such that $F_A \subseteq H_A$ and $sscl(H_A) \subseteq G_A$.

Proof. Let (U_A, τ) be seminormal space and F_A be a semiclosed soft set and G_A be a semiopen soft set containing $F_A \Rightarrow F_A$ and $(G_A)^c$ are disjoint semiclosed soft sets $\Rightarrow \exists$ two disjoint semiopen soft sets H_{A_1}, H_{A_2} such that $F_A \subseteq H_{A_1}$ and $(G_A)^c \subseteq H_{A_2}$. Now $H_{A_1} \subseteq (H_{A_2})^c \Rightarrow sscl H_{A_1} \subseteq sscl(H_{A_2})^c = (H_{A_2})^c$. Also, $(G_A)^c \subseteq H_{A_2} \Rightarrow (H_{A_2})^c \subseteq G_A \Rightarrow sscl H_{A_1} \subseteq (G_A)$.

Conversely, let L_A and K_A be any disjoint pair semiclosed soft sets $\Rightarrow L_A \subseteq (K_A)^c$, then by hypothesis there exists an semiopen soft set H_A such that $L_A \subseteq H_A$ and $sscl H_A \subseteq (K_A)^c \Rightarrow (K_A) \subseteq (sscl H_A)^c \Rightarrow (H_A)$ and $(sscl H_A)^c$ are disjoint semiopen soft sets such that $L_A \subseteq H_A$ and $K_A \subseteq (sscl H_A)^c$. \square

Theorem 6.16. Let $f : (U_A, \tau) \rightarrow (U_B, \delta)$ be a soft surjective function which is both irresolute and soft semiopen. If U_A is soft seminormal space then so is U_B .

Proof. Take a disjoint pair L_A and M_A of semiclosed soft sets of $U_B \Rightarrow f^{-1}(L_A)$ and $f^{-1}(M_A)$ are disjoint semiclosed soft sets of $U_A \Rightarrow \exists$ disjoint semiopen soft sets G_A and H_A such that $f^{-1}(L_A) \subseteq G_A$ and $f^{-1}(M_A) \subseteq H_A \Rightarrow L_A \subseteq f(G_A)$ and $M_A \subseteq f(H_A) \Rightarrow f(G_A)$ and $f(H_A)$ are disjoint open soft sets of U_B . \square

Theorem 6.17. *A semiclosed soft subspace of a soft seminormal space is soft seminormal.*

Proof. Let V_B be a semiclosed soft subspace of a soft seminormal space U_A . Take a disjoint pair L_B and M_B of semiclosed sets of $V_B \Rightarrow \exists$ disjoint semiclosed soft sets L_A and M_A such that $L_B = L_A \tilde{\cap} V_B, M_B = M_A \tilde{\cap} V_B$. Now by soft seminormality of U_A , \exists disjoint semiopen soft sets G_A and H_A such that $L_A \tilde{\subset} G_A$ and $M_A \tilde{\subset} H_A \Rightarrow L_B \tilde{\subset} G_A \tilde{\cap} V_B$ and $M_B \tilde{\subset} H_A \tilde{\cap} V_B$ \square

Theorem 6.18. *Every soft semicompact semi T_2 - space is seminormal.*

Proof. Let (U_A, τ) be a semicompact semi T_2 - space. Take a disjoint pair L_A and M_A of semiclosed sets. By theorem 6.12, for each soft point e_L , \exists disjoint semiopen soft sets G_{e_L} and H_{e_L} such that $e_L \tilde{\subset} G_{e_L}$ and $M_A \tilde{\subset} H_{e_L}$. So the collection $\{G_{e_{L_i}} \mid e_L \tilde{\subset} G_{e_{L_i}}, i \in \Lambda\}$ is a semiopen cover of G_{e_L} . Then by theorem 4.7, \exists a finite subfamily $\{G_{e_{L_i}} \mid i = 1, 2, \dots, n\}$ such that $G_{e_L} \tilde{\subset} \bigcup \{G_{e_{L_i}} \mid i = 1, 2, \dots, n\}$. Take $G_A = \bigcap \{G_{e_{L_i}} \mid i = 1, 2, \dots, n\}$ and $H_A = \bigcap \{H_{e_{L_i}} \mid i = 1, 2, \dots, n\}$. Then G_A and H_A are disjoint semiopen soft sets such that $L_A \tilde{\subset} G_A$ and $M_A \tilde{\subset} H_A$. Hence U_A is seminormal. \square

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